VOIMAN MOMENTTI *ristitulona*

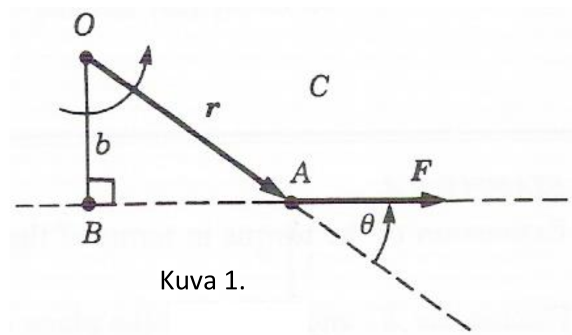
VOIMAN MOMENTTI = voiman kiertovaikutus (voima x voiman varsi);

$$\boxed{M = F \cdot d}$$

yksikkö: [M] = Nm

Voiman varsi = voiman vaikutussuoran kohtisuora etäisyys kiertoakselista O.

<u>yleisesti:</u> momentti on vektorisuure, joka määritellään *ristitulona:* $\overline{M} = \overline{r} \times \overline{F}$, $M = F_{\perp} \cdot r$ (MAOL s. 126 (118)).



Kuva 1.

Viereisen kuviossa hiukkaseen A vaikuttaa voima F, joka saa aikaan kappaleen A kiertymisen kiertoakselin O ympäri. Paikkavektori r on kappaleen A etäisyys kiertoakselista. Kulma θ on paikkavektorin r ja voiman F vaikutussuoran välinen kulma ja b on voiman F varsi eli voiman F vaikutussuoran kohtisuora etäisyys kiertoakselista O. Voiman F vääntövaikutusta kuvaava suure eli voiman F momentti pisteen O suhteen on **M = Fb = Frsinθ** $= F_{\perp} \cdot r$. Tässä voiman F varsi b = rsinθ. Momenttivarren b sijaan voidaan laskea myös voiman F kohtisuorana projektio kiertoakselia OA = r vastaan, jolloin $F_{\perp} = F\sin\theta$ ja $M = F_{\perp} \cdot r$.

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Voiman momentti määritellään siis yleisesti ristitulona eli vektoritulona $\bar{M} = \bar{r} \times \bar{F}$, joka on vektori, mikä on kohtisuorassa vektorien $\bar{r}$ ja $\bar{F}$ määrittämää tasoa vastaan (MAOL s. 35,126 (41,118)).Momentin suuruus eli momenttivektorin pituus (itseisarvo) on **M = Fb = Frsinθ**, missä sinθ = b/r (ks. kuva 1).

Voiman momentti	M	Nm	$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, $M = F_{\perp} r$

MAOL s.126 (118)

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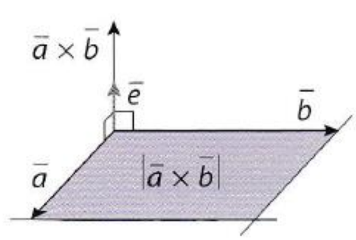
ristitulo (vektoritulo)

$$\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin(\bar{a}, \bar{b})\bar{e} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}, \ |\bar{e}| = 1$$



MAOL s.35(41)

8.3 Torque

When a force acts on a body, the body does not merely move in the direction of the force but it also usually turns about some point. Consider a force $\boldsymbol{F}$ acting on a particle A (Figure 8.6). Suppose that the effect of the force is to move the particle around O. Our daily experience suggests that the rotating effectiveness of $\boldsymbol{F}$ increases with the perpendicular distance (called **lever arm**) $b = OB$ from the center of rotation O to the **line of action** of the force. For example, when we open a door, we always push or pull as far as possible from the hinges and attempt to keep the direction of our push or pull perpendicular to the door. This common experience therefore suggests the convenience of defining a physical quantity τ that will be called **torque**, according to

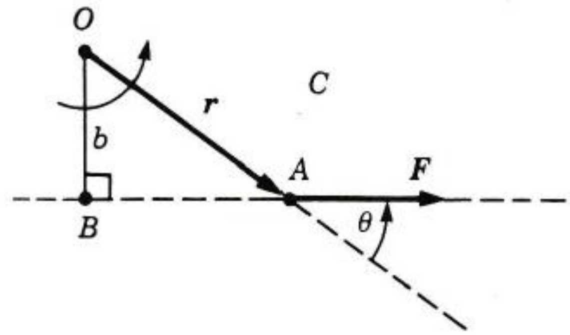


Figure 8.6 Torque of a force.

$$\tau = Fb \qquad \text{or} \qquad \text{torque} = \text{force} \times \text{lever arm}$$

Torque must be expressed as the product of a *unit of force* and a *unit of distance*. Thus in the SI system, torque is expressed in newton meters or N m.

Noting from the figure that $b = r \sin \theta$, we also may write

$$\tau = Fr \sin \theta \tag{8.5}$$

Comparing this equation with Equation A.21 for the vector product, we conclude that the torque may be considered as a vector quantity given by the vector product

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} \tag{8.6}$$

where $\boldsymbol{r}$ is the position vector, relative to O, of the point A on which the force is acting.

According to the properties of the vector product, the torque is represented by a vector perpendicular to both $\boldsymbol{r}$ and $\boldsymbol{F}$. That is, the torque is perpendicular to the plane drawn through both $\boldsymbol{r}$ and $\boldsymbol{F}$, and directed according to the right-hand rule (Figure 8.7).

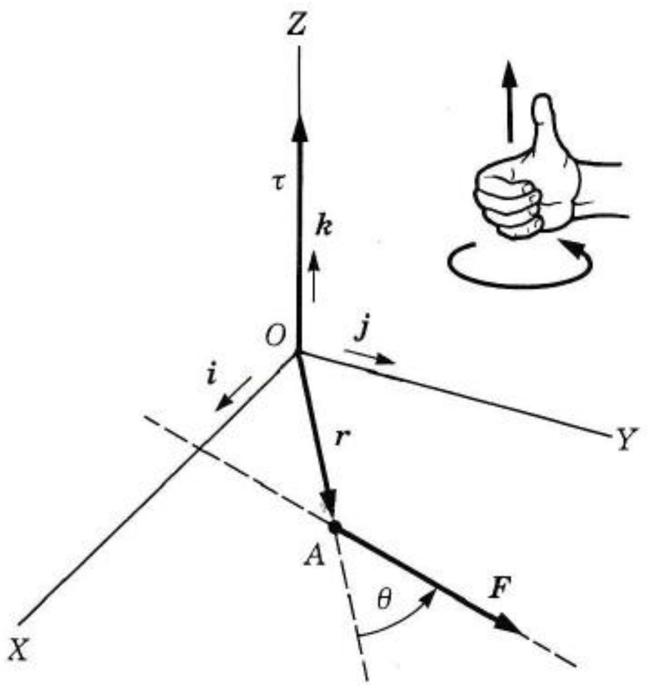


Figure 8.7 The torque is perpendicular to the plane determined by $\boldsymbol{r}$ and $\boldsymbol{F}$.

EXAMPLE 8.4

Expression of the torque in terms of the components of $\boldsymbol{r}$ and $\boldsymbol{F}$.

▷ Placing the X- and Y-axes in the plane determined by $\boldsymbol{r}$ and $\boldsymbol{F}$ (Figure 8.7), we have that

$$\boldsymbol{r} = \boldsymbol{i}x + \boldsymbol{j}y \qquad \text{and} \qquad \boldsymbol{F} = \boldsymbol{i}F_x + \boldsymbol{j}F_y$$

where (x, y) are the coordinates of A and (F_x, F_y) are the components of $\boldsymbol{F}$. Hence, $\boldsymbol{\tau} = (\boldsymbol{i}x + \boldsymbol{j}y) \times (\boldsymbol{i}F_x + \boldsymbol{j}F_y)$, and by application of Equation A.24 we obtain

$$\boldsymbol{\tau} = \boldsymbol{k}(xF_y - yF_x) \tag{8.7}$$

which is a vector parallel to the Z-axis, as illustrated in Figure 8.7. The magnitude of the torque is

$$\tau = xF_y - yF_x \tag{8.8}$$

The torque is positive or negative depending on the sense of rotation around the Z-axis.

Marcelo Alonso & Edward J. Finn: Physics, Addison-Wesley, 1992, pp. 145-146.